CBSE Board

Class X Mathematics (Standard)

Sample Paper - 1

Term 2 – 2021 - 22

Time: 2 hours **Total Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section A

Q1 – Q6 are of 2 mark each.

1. Which term of the AP 3, 8, 13, 18, … is 78?

OR

Find the 20^{th} term from the last term of the AP 3, 8, 13, ……, 253

2. Find the roots of the following quadratic equation $x^2 - 3x - 10 = 0$ by factorisation

3. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

4. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboids.

5. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

6. Find two numbers whose sum is 27 and product is 182.

OR

Find two consecutive positive integers, sum of whose squares is 365.

Section B

Q6 – Q10 are of 3 mark each.

7. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

8. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

9. Find the following tables gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	**Number of lamps**
1500 – 2000	14
2000 – 2500	56
2500 – 3000	60
3000 – 3500	86
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

Find the median life time of a lamp.

10. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

OR

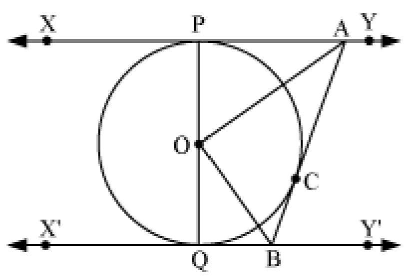
A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Section C

Q11 – Q14 are of 4 mark each.

11. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use $\pi = 3.14$)

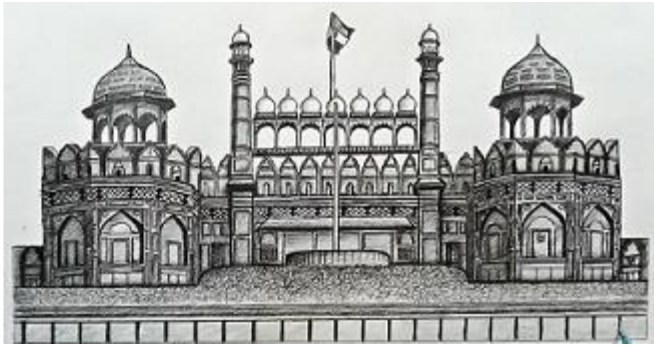
12. In the given figure XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY and A and X′Y′ at B. Prove that $\angle AOB = 90^\circ$.



OR

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

13. Sanjana took her students of class X to an educational trip where they saw Red Fort. She told them about the history of Red Fort where she narrated that Red Fort or Lal Qila is a historic fort located in Old Delhi, India that served as the main residence of the Mughal Emperors. Emperor Shah Jahan commissioned construction of the Red Fort on 12 May 1638, when he decided to shift his capital from Agra to Delhi. She also included that the Red Fort is about 18–33 m (59 – 108 ft) high.



Then answer the following questions.

i) Draw a labelled figure on the basis of the given information and find the angle of elevation if the maximum height of Red fort is considered and the students are standing at a distance of 33m away from the Monument.

ii) Draw a labelled figure and find the height of the tower if it casts a shadow of 30 m and the rays of the Sun is inclined at 30^{o}.

14. Nirvana's father starts a new footwear shop. To display the foot wears, he puts 3 pairs of sandals in 1st row and increases the number of pairs in subsequent rows by 2. Now based on the given information, answer the following questions.

i) Form an A.P representing the number of pairs of shoes and hence find the minimum number of rows required to store 120 pairs.

ii) If he is able to sell all the footwear accept for rows 14th and 7th. Then, find the total number of pairs available in the shop.

Solution

Section A

1.

3, 8, 13, 18, …
For this AP,
$a = 3$
$d = a_2 - a_1 = 8 - 3 = 5$
Let n^{th} term of this AP be 78.
$a_n = a + (n - 1)\ d$
$78 = 3 + (n - 1)\ 5$
$75 = (n - 1)\ 5$
$(n - 1) = 15$
$n = 16$
Hence, 16^{th} term of this AP is 78.

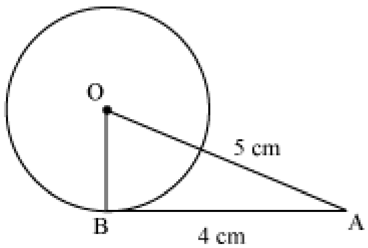
OR

3, 8, 13,….. …, 253
Common difference for this AP is 5.
Therefore, this AP can be written in reverse order as
253, 248, 243, …, 13, 8, 5
For this AP,
$a = 253$
$d = 248 - 253 = -5$
$n = 20$
$a_{20} = a + (20 - 1)\ d$
$a_{20} = 253 + (19)\ (-5)$
$a_{20} = 253 - 95$
$a = 158$
Therefore, 20^{th} term from the last term is 158.

2.

$$x^2 - 3x - 10$$
$$= x^2 - 5x + 2x - 10$$
$$= x(x-5) + 2(x-5)$$
$$= (x-5)(x+2)$$

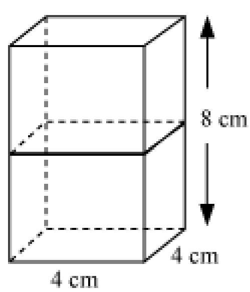
3.



AB is a tangent drawn to the circle, with centre O, from point A.
OA = 5cm and AB = 4 cm
Since, radius is perpendicular at the point of contact, OB $\perp$ AB.
Applying Pythagoras theorem in ΔABO,
$AB^2 + BO^2 = OA^2$
$4^2 + BO^2 = 5^2$
$BO^2 = 9$
BO = 3
Hence, the radius of the circle is 3 cm.

4.

Given that
Volume of cubes = 64 cm^3
$(\text{Edge})^3 = 64$
Edge = 4



If cubes are joined end to end, dimensions of resulting cuboid will be 4 cm, 4 cm, 8 cm.

$\therefore$ surface area and cuboids $= 2(lb + bh + lh)$

$$= 2(4 \times 4 + 4 \times 8 + 4 \times 8)$$
$$= 2(16 + 32 + 32)$$
$$= 2(16 + 64)$$
$$= 2 \times 80 = 160 \text{ cm}^2$$

5.

Let us find class marks (x_i) for each interval by using the relation.

Class mark (x_i) $= \dfrac{\text{upper class limit} + \text{lower class limit}}{2}$

Now we may compute x_i and f_ix_i as following

Number of plants	**Number of houses (f_i)**	x_i	f_ix_i
0 – 2	1	1	1 × 1 = 1
2 – 4	2	3	2 × 3 = 6
4 – 6	1	5	1 × 5 = 5
6 – 8	5	7	5 × 7 = 35
8 – 10	6	9	6 × 9 = 54
10 – 12	2	11	2 ×11 = 22
12 – 14	3	13	3 × 13 = 39
Total	20		162

From the table we may observe that

$\sum f_i = 20$

$\sum f_ix_i = 162$

Mean $\bar{x} = \dfrac{\sum f_ix_i}{\sum f_i}$

$= \dfrac{162}{20} = 8.1$

So, mean number of plants per house is 8.1.

6.

Let the first number be x and the second number is $27 - x$.
Therefore, their product $= x(27 - x)$
It is given that the product of these numbers is 182.

Therefore, $x(27-x)=182$

$\Rightarrow x^2-27x+182=0$

$\Rightarrow x^2-13x-14x+182=0$

$\Rightarrow x(x-13)-14(x-13)=0$

$\Rightarrow (x-13)(x-14)=0$

Either x - 13 = 0 or $x - 14 = 0$
i.e., $x = 13$ or $x = 14$
Therefore, the numbers are 13 and 14.

OR

Let the consecutive positive integers be x and $x + 1$

Given that $x^2+(x+1)^2=365$

$\Rightarrow x^2+x^2+1+2x=365$

$\Rightarrow 2x^2+2x-364=0$

$\Rightarrow x^2+x-182=0$

$\Rightarrow x^2+14x-13x-182=0$

$\Rightarrow x(x+14)-13(x+14)=0$

$\Rightarrow (x+14)(x-13)=0$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$
Since the integers are positive, x can only be 13.
$\therefore x + 1 = 13 + 1 = 14$
Therefore, two consecutive positive integers will be 13 and 14.

Section B

7.

From the data given as above we may observe that maximum class frequency is 61 belonging to class interval 60 – 80.
So, modal class = 60 – 80
Lower class limit (l) of modal class = 60
Frequency (f_1) of modal class = 61
Frequency (f_0) of class preceding the modal class = 52
Frequency (f_2) of class succeeding the modal class = 38
Class size (h) = 20

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{61 - 52}{2(61) - 52 - 38}\right)(20)$$

$$= 60 + \left(\frac{9}{122 - 90}\right)(20)$$

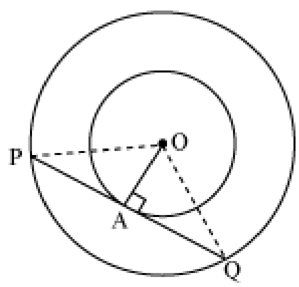
$$= 60 + \left(\frac{9 \times 20}{32}\right)$$

$$= 60 + \frac{90}{16} = 60 + 5.625$$

$$= 65.625$$

So, modal lifetime of electrical components is 65.625 hours.

8.



Let two concentric circles be centered at point O. Let PQ be the chord of the larger circle which touches the smaller circle at point A. So, PQ is tangent to smaller circle.

Since, OA is radius of circle, $OA \perp PQ$

Applying Pythagoras theorem in ΔOAP,

$OA^2 + AP^2 = OP^2$

$3^2 + AP^2 = 5^2$

$AP^2 = 16$

AP = 4 cm

In ΔOPQ, as $OA \perp PQ$, AP = AQ

(Perpendicular from center of circle bisects the chord)

$\therefore$ PQ = 2AP = 2 × 4 cm = 8 cm

So, length of chord of larger circle is 8cm.

9.

We can find cumulative frequencies with their respective class intervals as below -

Life time	**Number of lamps (f_i)**	**Cumulative frequency**
1500 – 2000	14	14
2000 – 2500	56	14 + 56 = 70
2500 – 3000	60	70 + 60 = 130
3000 – 3500	86	130 + 86 = 216
3500 – 4000	74	216 + 74 = 290
4000 – 4500	62	290 + 62 = 352
4500 – 5000	48	352 + 48 = 400
Total (n)	400	

Now we may observe that cumulative frequency just greater than

$\frac{n}{2}\left(\text{i.e. } \frac{400}{2} = 200\right)$ is 216 belonging to class interval 3000 – 3500.

Median class = 3000 – 3500

Lower limit (l) of median class = 3000

Frequency (f) of median class = 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 3000 + \left(\frac{200 - 130}{86}\right) \times 500$$

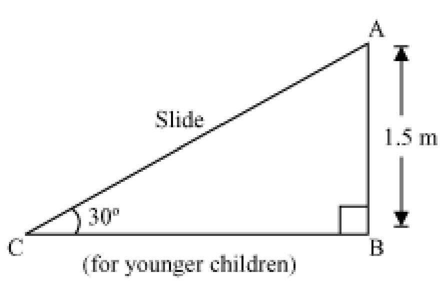
$$= 3000 + \frac{70 \times 500}{86}$$

$$= 3406.976$$

So, median life time of lamps is 3406.98 hours.

10.

In the two figures, AC and PR are the slides for younger and elder children respectively

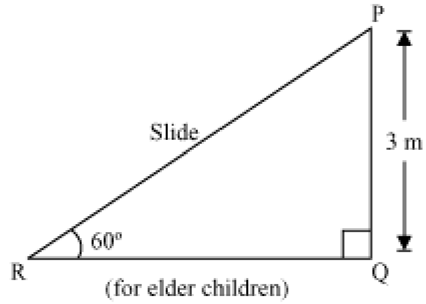


In ΔABC,

$\frac{AB}{AC} = \sin 30°$

$\frac{1.5}{AC} = \frac{1}{2}$

AC = 3 m



In ΔPQR,

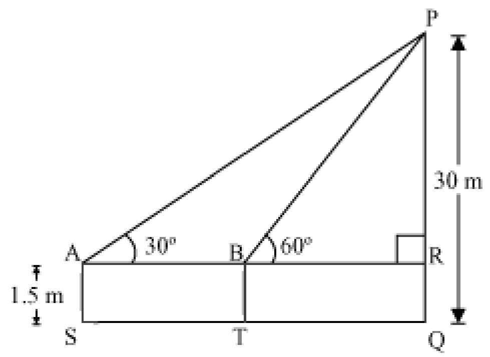
$\frac{PQ}{PR} = \sin 60$

$\frac{3}{PR} = \frac{\sqrt{3}}{2}$

$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ m

Thus, the lengths of the two slides were 3 m and $2\sqrt{3}$ m.

OR



Let the initial position of the boy be S. He walks towards building and reached at point T.

In the figure, PQ is the building of height 30 m.

AS = BT = RQ = 1.5 m

PR = PQ – RQ = 30 m – 1.5 m = 28.5

In ΔPAR,

$\frac{PR}{AR} = \tan 30^\circ$

$\frac{28.5}{AR} = \frac{1}{\sqrt{3}}$

$AR = 28.5\sqrt{3}$

In ΔPRB,

$\frac{PR}{BR} = \tan 60^\circ$

$\frac{28.5}{BR} = \sqrt{3}$

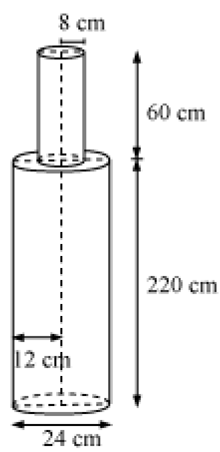
$BR = \frac{28.5}{\sqrt{3}} = 9.5\sqrt{3}$

$ST = AB = AR - BR = 28.5\sqrt{3} - 9.5\sqrt{3} = 19\sqrt{3}$

Thus, the distance which the boy walked towards the building is $19\sqrt{3}$ m.

Section C

11.



From the figure we have

Height (h_1) of larger cylinder = 220cm

Radius (r_1) of larger cylinder = $\frac{24}{2}$ = 12cm

Height (h_2) of smaller cylinder = 60cm

Radius (r_2) of larger cylinder = 8cm

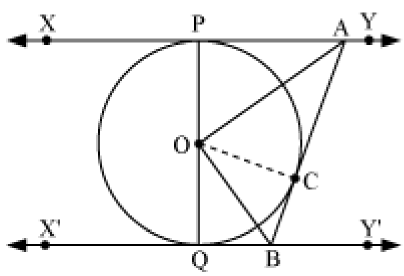
Total volume of pole $=$ volume of larger cylinder $+$ volume of smaller cylinder

$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$
$$= \pi(12)^2 \times 220 + \pi(8)^2 \times 60$$
$$= \pi[144 \times 220 + 64 \times 60]$$
$$= 35520 \times 3.14 = 1,11,532.8 \text{ cm}^3$$

Mass of 1 cm^3 iron = 8gm

Mass of 111532.8 cm^3 iron = 111532.8 × 8 = 892262.4 gm = 892.262 kg.

12.



Join OC.

In ΔOPA and ΔOCA,

OP = OC (Radius of the same circle)

AP = AC (tangents from point A)

AO = AO (common)

$\Delta OPA \cong \Delta OCA$ (SSS congruence rule)

$\therefore \angle POA = \angle COA$ … (1)

Similarly $\Delta OQB \cong \Delta OCB$

$\therefore \angle QOB = \angle COB$ … (2)

Since POQ is a diameter of circle we can say it is a straight line.

So, $\angle POA + \angle COA + \angle COB + \angle QOB = 180°$

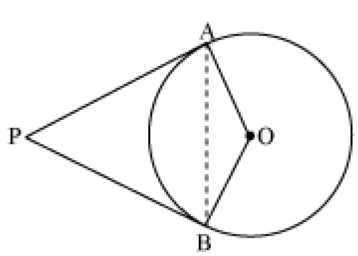
Now from equations (1) and (2),

$2\angle COA + 2\angle COB = 180°$

$(\angle COA + \angle COB) = 90°$

$\angle AOB = 90°$

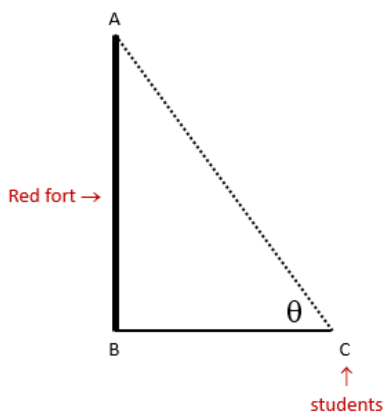
OR



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to circle which are touching circle at point A and B respectively.
AB is the line segment, joining point of contacts A and B together such that it subtends
$\angle$AOB at center O of circle.
As the radius is perpendicular to the tangent at the point of contact, $\angle$OAP = 90°.
Similarly, $\angle$OBP = 90°
In quadrilateral OAPB,
$\angle$OAP + $\angle$APB + $\angle$PBO + $\angle$BOA = 360°
90° + $\angle$APB + 90° + $\angle$BOA = 360°
$\angle$APB + $\angle$BOA = 180°
Hence, the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

13.

i)

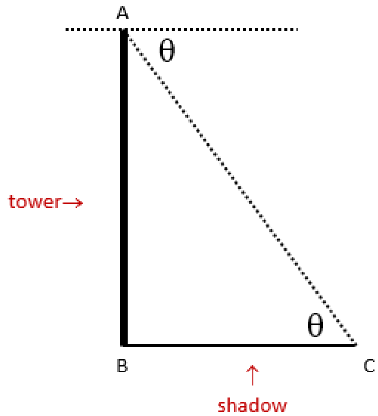


Let AB represents Red Fort with height 33m and BC be the distance.
Let θ be the angle of elevation.
Using trigonometry, we have

$$\tan\theta = \frac{AB}{BC} = \frac{33}{33} = 1$$

$$\Rightarrow \theta = 45^\circ$$

ii)



Let AB be the height and BC be the shadow casted by AB.

$\theta = 30^o$

$\tan 30^o = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$

$\Rightarrow AB = 10\sqrt{3}$ m

14.

i) AP is 2,5,7,9,11,13,....

The number of pairs put in the rows form an A.P. with first term 3 and common difference 2.

Total number of pairs = 120

$120 = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow n\left(6 + (n-1)2\right) = 240$

$\Rightarrow n(n+2) = 120$

$\Rightarrow n^2 + 2n - 120 = 0$

$\Rightarrow (n+12)(n-10) = 0$

As the number of rows can't be negative.

Therefore, the number of rows is 10.

ii) nth term of AP is $a_n = a + (n-1)d$

14th row = a + 13d = 3 + 26 = 29

7th row = a + 6d = 3 + 12 = 15

Total number of pairs available = 29 + 15 = 44